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Strings and D-branes in a supersymmetric magnetic flux background

Roberto Iengo,[a] Jaume Lopez Carballo[b] and Jorge G. Russo[bc]

[a] *International School for Advanced Studies (SISSA)*
Via Beirut 2-4, I-34013 Trieste, Italy, and
INFN, Sezione di Trieste
[b] *Departament ECM, Facultat de Física, Universitat de Barcelona,*
Diagonal, 647, 08028 Barcelona, Spain
[c] *Institució Catalana de Recerca i Estudis Avançats (ICREA)*
Passeig Llus Companys, 23, 08010 Barcelona, Spain
E-mail: jrusso@ub.edu

Abstract: We investigate how the presence of RR magnetic F_{p+2} fluxes affects the energy of classical Dp branes, for specific string theory supersymmetric backgrounds which are solutions to the leading order in α' including back-reaction effects. The Dp brane dynamics is found to be similar to the well known dynamics of particles and strings moving in magnetic fields. We find a class of BPS solutions which generalize the BPS fundamental strings or BPS branes with momentum and winding to the case of non-zero magnetic fields. Remarkably, the interaction with the magnetic fields does not spoil the supersymmetry of the solution, which turns out to be invariant under four supersymmetry transformations. We find that magnetic fields can significantly reduce the energy of some BPS strings and Dp branes, in particular, some macroscopic Dp branes become light for sufficiently large magnetic fields.

Keywords: Brane Dynamics in Gauge Theories, D-branes, Conformal Field Models in String Theory.

Contents

1. Introduction

Flux compactifications in string theory stabilize the moduli and presently provide the main framework to connect strings to the real world [1, 2]. The fluxes induce a non-trivial potential energy for the moduli. In addition, the fluxes can significantly affect the energy spectrum of some quantum states. The precise modification of the energy of a given quantum state depends on the particular string compactification, but there are some general features which are part of the well known physics of particles interacting with magnetic fields. In quantum theory, the energy of a state with electric charge e and spin S moving in a magnetic field is given by the formula

$$E^2 = k_i^2 + 2eB_z\left(l + \frac{1}{2} - S_z\right), \qquad l = 0, 1, 2, \dots\, , \quad -S \leq S_Z \leq S\ . \tag{1.1}$$

For states with spin aligned with the magnetic fields, the energy is reduced and the state can even become tachyonic for certain values of the magnetic field, leading to instabilities.

A natural question is whether analogous instabilities could be present in superstring theory in spaces containing NSNS or RR magnetic fluxes turned on in some directions. In the context of type II string theory, there are exactly solvable models where the magnetic field originates by Kaluza-Klein reduction from the metric or from the antisymmetric tensor $B_{\mu\nu}$ [3]. These models include gravitational back reaction effects due to the energy density of the magnetic field, and exhibit tachyonic instabilities for magnetic fields greater than some critical values [3 − 5].

It is possible to consider similar closed string models with magnetic field configurations that preserve supersymmetry [6]. In this case the physical string spectrum is tachyon free. Nevertheless, one can expect that for certain values of the magnetic field, the energy can be significantly reduced due to a negative contribution from the gyromagnetic interaction, so that a macroscopic string of size $\gg l_s$ can even become light, $E \sim l_s^{-1}$. In this paper we will show that this is indeed the case and study the analog phenomenon for Dp branes. To this aim, we will consider a string-theory background with a RR magnetic F_{p+2} flux which is obtained by S and T-dualities from the magnetic background found in [6].

This background has two magnetic parameters B_1, B_2. When $B_1 = B_2$, the background preserves 1/2 of the 32 supersymmetries. We will find classical string and Dp brane configurations becoming light for certain values of the magnetic field parameters B_1, B_2. Remarkably, these solutions are BPS, despite the interaction with the magnetic field. The solutions preserve a fraction 1/4 of the 16 supersymmetries of the background and therefore are invariant under 4 supersymmetry transformations.

Studies of Dp brane classical solutions in flat space can be found for example in [7] and more recently [8]. Different studies of the conditions to have supersymmetric Dp branes in some supersymmetric compactifications with Ramond-Ramond (RR) fluxes are in [9 − 11].

The organization of this paper is as follows. In section 2.1 we review the string spectrum in a particular flat (but globally non-trivial) background which gives rise to a magnetic field by Kaluza-Klein reduction; we recall the supersymmetry properties of the background and the presence of tachyons when supersymmetry is broken (the main points of this quantum analysis are reviewed in the appendix A). In section 2.1.1 we identify BPS states for the supersymmetric background with $B_1 = B_2$. In section 2.2 we construct a family of classical solutions which corresponds to BPS states by solving the classical equations of motion. In section 2.3 we show that these classical solutions indeed preserve a fraction of supersymmetry, by using both cartesian and polar coordinates. In section 3.1 we study the background obtained by a T-duality transformation; this background contains explicitly a $B_{\mu\nu}$ field and the metric is curved due to the back reaction produced by the magnetic energy density. The spectrum, and in particular the BPS states, are obtained from section 2.1 by the standard T-duality rules. In section 3.2 we consider a family of classical solutions corresponding to the BPS states by solving the classical equations in the background of section 3.1 by using the Polyakov formalism. In section 3.3 we repeat this computation using the Nambu-Goto formalism; this is done in preparation for the study of classical Dp-branes solutions, whose dynamics is governed by the Dirac-Born-Infeld action. In fact, in the case of the Dp-branes, studied in section 4, the method and the various steps for obtaining the solutions will be seen to be essentially the same. We find a family of BPS classical solutions, the energy spectrum being a generalization of the BPS spectrum in the magnetic background discussed in section 3. One important feature of the result

JHEP08(2007)047

is the presence of light states which have macroscopic features. This result is further discussed in section 5. Finally, in the appendix B we consider another family of classical F-string solutions in the background of section 3 (or, equivalently, D-string solutions in the background of section 4). These solutions, although not BPS, are nevertheless interesting because they generalize to a non-flat background with fluxes the widely studied case of the folded rotating string.

2. Fundamental string in magnetic backgrounds

2.1 Magnetic fields from Kaluza-Klein reduction

We shall consider a simple magnetic string model given in terms of the background [6]

$$ds^2 = -dt^2 + dx_s^2 + dy^2 + dr_1^2 + r_1^2(d\varphi_1 + B_1 dy)^2 + dr_2^2 + r_2^2(d\varphi_2 + B_2 dy)^2 \,, \tag{2.1}$$

where all other supergravity fields are trivial and $s = 6, \dots, 9$. This is an exact conformal string model, since the metric is flat. It is globally non-trivial, due to the fact that y is a periodic coordinate, $y = y + 2\pi R$.

The background preserves 1/2 of the 32 supersymmetries provided

$$B_1 = \pm B_2 \ . \tag{2.2}$$

The string model is a simple generalization of the non-supersymmetric string model with $B_2 = 0$ that was solved in [3]. The exact physical string spectrum is found in a similar way and it is given by [6]

$$\begin{aligned} \alpha' M^2 &= 2(N_L + N_R) + \alpha'\left(\frac{n}{R} - B_1 J_1 - B_2 J_2\right)^2 + \frac{m^2 R^2}{\alpha'} \\ &\quad - 2B_1 Rm(J_{1R} - J_{1L}) - 2B_2 Rm(J_{2R} - J_{2L})\,, \\ N_R - N_L &= mn\,, \end{aligned} \tag{2.3}$$

where B_1 and B_2 are in the interval $0 \leq \gamma_{1,2} < 1$, $\gamma_{1,2} \equiv B_{1,2} Rm$. For other intervals the spectrum is repeated periodically in the parameters $\gamma_{1,2}$ with period 1. The angular momentum operators for the two planes (r_1, φ_1) and (r_2, φ_2) are given by

$$J = J_R + J_L\,, \qquad J_{L,R} = \pm\left(l_{L,R} + \frac{1}{2}\right) + S_{L,R}\,, \tag{2.4}$$

where for the sake of clarity we omitted the obvious indices 1,2. $N_{L,R} = 0, 1, 2, \dots$. are the standard excitation number operators of flat-space type II superstring theory and $S_{L,R}$ are the standard Left and Right contributions to the flat-space spin operator (the main points of the derivation are reported in the appendix A, see [3] for details). The parameters $l_{L,R} = 0, 1, 2, \dots$ are orbital angular momenta (Landau numbers). The parameters m, n represent winding and momentum in the y direction. If there are other compact coordinates among the x_s, then their winding and momentum contributions to the energy is added in the standard way as in the flat case. The spin operators satisfy the inequalities

$$\left|S_{1L,R} + S_{2L,R}\right| \leq N_{L,R} + 1\,, \qquad \left|S_{1L,R} - S_{2L,R}\right| \leq N_{L,R} + 1 \ . \tag{2.5}$$

One finds that $M^2 \geq 0$ for $B_1 = \pm B_2$ (see appendix A), but The spectrum contains tachyons in some regions of the parameter space [3, 6], if $B_1 \neq \pm B_2$. For example, consider a state with the following quantum numbers:

$$\begin{aligned} N_R = N_L = 0\,, \quad & S_{1R} = -S_{1L} = 1\,, \quad S_{2R} = S_{2L} = 0\,, \\ l_{1L,R} = l_{2L,R} = 0\,, \quad & m = 1\,, \qquad\qquad n = 0\,, \end{aligned} \tag{2.6}$$

so that $J_1 = J_2 = 0\,,\; J_{1R} - J_{1L} = 1\,,\; J_{2R} - J_{2L} = -1$ and

$$\alpha' M^2 = \frac{R^2}{\alpha'} - 2(B_1 - B_2)R\ . \tag{2.7}$$

The state becomes tachyonic for $B_1 - B_2 > R/(2\alpha')$. In the case $B_1 = B_2$, the full spectrum is tachyon free for any B_1, consistently with supersymmetry.

Another example is a state classically identical to the BPS state discussed below, see sections 2.1.1 and 2.2. In appendix A we show that, for instance for $B_2 = 0$, this state can have $M^2 < 0$ in some parameters range. For $B_2 = B_1$ (taking here both positive) the positive zero point energy of the Landau levels in the plane 2 (a quantum effect of the sigma model) insures that $M^2 \geq 0$.

Therefore the classical analysis, that would give M^2 as a sum of contributions ≥ 0, is not enough to guaranty the absence of tachyons, if supersymmetry is completely broken. However, this occurs when in parameters regions where the classical result for the mass is microscopic, that is of the order of the inverse string length or of the inverse compactification radius.

Most of the present paper is devoted to the study of states having macroscopic features but microscopic mass. Those are precisely the cases which could give rise to tachyonic instabilities, if they are not protected by supersymmetry.

In [12, 13] the D brane spectra in the background (2.1) has been determined by using the boundary state formalism, exhibiting a number of interesting features (the orientifold spectrum was studied in [14]). In section 4, instead, we will be interested in studying Dp branes not in (2.1), but in backgrounds containing F_{p+2} fluxes, which are obtained from (2.1) by dualities. The spectrum, and the physics in general, are clearly different in each case, the former [12, 13] being analogous to a neutral particle (or zero-winding string) moving in the geometry (2.1) (and on the curved-space generalization discussed in section 3), whereas the latter (section 4) involves a physics which is similar to that of a charged particle moving in a magnetic field.

2.1.1 BPS states

In the $B_1 = B_2 = 0$ case, an important class of quantum string states are the BPS string states with [15]

$$N_L = 0\ , \qquad N_R = mn\ . \tag{2.8}$$

Then M^2 becomes a perfect square:

$$\alpha' M^2_{\rm BPS} = 2mn + \alpha' \frac{n^2}{R^2} + \frac{m^2 R^2}{\alpha'} = \alpha' \left(\frac{n}{R} + \frac{mR}{\alpha'} \right)^2 , \tag{2.9}$$

or

$$M_{\rm BPS} = \left| \frac{n}{R} + \frac{mR}{\alpha'} \right| \ . \tag{2.10}$$

Note that $mn = N_R > 0$. When $mn < 0$, there are BPS states with $N_R = 0$, $N_L = -mn$ for which

$$M_{\rm BPS} = \left| \frac{n}{R} - \frac{mR}{\alpha'} \right| = \left| \frac{n}{R} \right| + \left| \frac{mR}{\alpha'} \right| \ . \tag{2.11}$$

We now look for similar supersymmetric states in the presence of magnetic fields B_1, B_2. We will restrict to the case $B_1 = B_2 \equiv B$ where the background preserves 16 supersymmetries. The state (2.8) has the energy

$$\begin{aligned} \alpha' M^2 = 2mn + \alpha' \frac{n^2}{R^2} + \frac{m^2 R^2}{\alpha'} - 2B\alpha' \frac{n}{R}(J_1 + J_2) - 2BmR(J_{1R} + J_{2R} - J_{1L} - J_{2L}) \\ + \alpha' B^2 (J_1 + J_2)^2 \ . \end{aligned} \tag{2.12}$$

This becomes a perfect square if $J_1 + J_2 = J_{1R} + J_{2R} - J_{1L} - J_{2L}$, i.e.,

$$S_{1L} + S_{2L} = -1 - l_{1L} - l_{2L} \ . \tag{2.13}$$

Since $N_L = 0$, $S_{1,2L}$ can be zero or ± 1 and $|S_{1L} + S_{2L}| \leq 1$. This implies $l_{1L} = l_{2L} = 0$ and the following possible values:

$$(S_{1L}, S_{2L}) = (-1, 0) \ \text{ or } \ (0, -1) \ . \tag{2.14}$$

Then we get

$$M^2_{\rm BPS} = \left(\frac{n}{R} + \frac{mR}{\alpha'} - B(J_1 + J_2) \right)^2 \ . \tag{2.15}$$

Remarkably, the mass square is a perfect square which indicates that the state is still supersymmetric despite the presence of the magnetic fields. It is worth noting that in the case $B_1 \neq B_2$, M^2 is not a perfect square. Moreover, the state can become tachyonic above some critical magnetic field.

Assume for definiteness $B > 0$, $m, n > 0$. Taking into account the fact that $mRB < 1$ and $(J_1 + J_2) \leq N_R = mn$, one can see that $\frac{n}{R} - B(J_1 + J_2) > 0$ and therefore eq. (2.15) can be written as

$$M_{\rm BPS} = \frac{|m|R}{\alpha'} + \left| \frac{n}{R} - B(J_1 + J_2) \right| \ . \tag{2.16}$$

This form will be suitable for comparison with the energy of classical strings.

The effect of the magnetic field on a quantum state with spin aligned with the magnetic field is to reduce its energy. The maximum reduction is attained for the state with maximum spin $S_{1R} + S_{2R} = N_R + 1$ and $l_{1,2R} = 0$, so that $J_1 + J_2 = N_R = nm$. For this state

$$M_{\rm BPS} = \frac{|m|R}{\alpha'} + \left| \frac{n}{R} - Bmn \right| \ . \tag{2.17}$$

The minimum mass occurs when B approaches the limit of the interval, $BRm \to 1$, where

$$M_{\rm BPS} \to \frac{|m|R}{\alpha'} \ . \tag{2.18}$$

JHEP08(2007)047

Assuming that R is of the same order of magnitude as α', this implies that there are states with very large n and small m which become light (mass of $O(R/\alpha')$) at some magnetic field. Such states have a macroscopic mass $= O(n/R) \gg \frac{1}{\sqrt{\alpha'}}$ in the absence of magnetic fields.

This result can be understood from the periodicity of the spectrum: the full quantum string spectrum at $BRm \to 1$ is the same as the spectrum at $B = 0$, with some relabelling of the states. In the coordinate system where the metric becomes Minkowski at $B = 0$, a given large classical string becomes very light, $E \sim 1/l_s$, as BRm approaches 1. But when $BRm \to 1$ the metric approaches the Minkowski metric in another coordinate system where the polar coordinate is $\varphi'_{1,2} = \varphi_{1,2} + y/mR$. The energy (2.18) then corresponds to a state with $n' = 0$ and orbital angular momentum (see section 2.2).

2.2 Classical BPS solution

There is an exponential number $= O\big(\exp(2\sqrt{2}\pi\sqrt{N_R})\big)$ of quantum states satisfying the condition $N_R = mn$, $N_L = 0$. A particular class of classical solutions representing a small subset of these states is given by

$$\begin{aligned} Z_1 &= r_1 e^{i\varphi_1} = X_1 + iX_2 = L_1\ e^{ik_1\sigma + i\omega_1\tau}\,, \\ Z_2 &= r_2 e^{i\varphi_2} = X_3 + iX_4 = L_2\ e^{ik_2\sigma + i\omega_2\tau}\,, \\ t &= \kappa\tau\,, \\ y &= mR\sigma + q\tau\ . \end{aligned} \tag{2.19}$$

where $0 \leq \sigma < 2\pi$ and $m,\ k_1,\ k_2$ are integer numbers. When $k_1 = k_2 = 1$, these solutions represent the states with maximum angular momentum having $S_{1R} + S_{2R} = N_R + 1$.

In section 2.3, we will show that these solutions are indeed supersymmetric. Classically, the condition (2.14) does not appear, since it arises due to normal ordering terms. The classical string description applies when $N_R, J_R \gg 1$ and $S_{1,2L} = -1$ becomes negligible.

In this subsection we will reproduce the energy (2.16) of the quantum string states for the cases of the circular BPS ("chiral") string (2.19). The Polyakov action is given by

$$\begin{aligned} S = -\frac{1}{4\pi\alpha'}\int d\sigma d\tau\Big(& -\partial_\alpha t\partial_\alpha t + \partial_\alpha y\partial_\alpha y + r_1^2\big(\partial_\alpha\varphi_1 + B_1\partial_\alpha y\big)\big(\partial_\alpha\varphi_1 + B_1\partial_\alpha y\big) \\ & + r_2^2\big(\partial_\alpha\varphi_2 + B_2\partial_\alpha y\big)\big(\partial_\alpha\varphi_2 + B_2\partial_\alpha y\big)\Big)\ . \end{aligned} \tag{2.20}$$

Here the indices α are contracted with the world-sheet metric $h_{\alpha\beta} = \mathrm{diag}(-1,1)$. Then the string equations of motion are automatically satisfied provided

$$(k_a + B_a mR) = \pm(\omega_a + B_a q)\,, \qquad a = 1,2\,, \tag{2.21}$$

which follow from the r_a equations of motion assuming that both L_1, L_2 are different from zero.

In addition, from the Virasoro constraints, we get the following conditions

$$0 = mRq + \sum_{a=1}^{2} L_a^2(k_a + B_a mR)(\omega_a + B_a q), \tag{2.22}$$

$$\kappa^2 = m^2R^2 + q^2 + \sum_{a=1}^{2} L_a^2\Big((k_a + B_a mR)^2 + (\omega_a + B_a q)^2\Big) . \tag{2.23}$$

Combining these equations, we find

$$\kappa^2 = (mR \pm q)^2 + \sum_{a=1}^{2} L_a^2\Big((k_a + B_a mR) \pm (\omega_a + B_a q)\Big)^2 . \tag{2.24}$$

Using (2.21), this gives

$$\kappa^2 = (mR \pm q)^2 , \tag{2.25}$$

where the plus sign holds for the solution with $(k_a + B_a mR) = -(\omega_a + B_a q)$ (the "Right" circular string) whereas the minus sign holds for the solution with $(k_a+B_a mR) = (\omega_a+B_a q)$ (the "Left" circular string).

Now we would like to express the energy in terms of the physical conserved quantum numbers $J_1,\ J_2$ and m, n. We have

$$\begin{aligned} E &= \int_0^{2\pi} d\sigma \frac{\delta S}{\delta(\partial_\tau t)} = -\frac{1}{\alpha'}\ \kappa\,, \\ J_a &= \int_0^{2\pi} d\sigma \frac{\delta S}{\delta(\partial_\tau \varphi_a)} = \frac{L_a^2}{\alpha'}(\omega_a + B_a q)\,, \\ \frac{n}{R} &= \int_0^{2\pi} d\sigma \frac{\delta S}{\delta(\partial_\tau y)} = \frac{1}{\alpha'}\Big(q + L_1^2 B_1(\omega_1 + B_1 q) + L_2^2 B_2(\omega_2 + B_2 q)\Big) . \end{aligned} \tag{2.26}$$

Using (2.25) we find

$$E = \frac{|m|R}{\alpha'} + \left|\frac{n}{R} - B_1 J_1 - B_2 J_2\right| , \tag{2.27}$$

reproducing exactly the result (2.16) of the quantum string spectrum, upon setting $B_1 = B_2$.

As mentioned in the previous subsection, for some parameters, there are states that have macroscopic features but have microscopic energy. Take for instance a string in the plane 1 that is $L_2 = J_2 = 0$ with $m = 1$, n large and $mRB_1 = 1 - 1/n$, $m, n > 0$ (inside the periodicity interval). Choosing $k_1 = -1$ and using (2.21) and (2.26) we get $J_1 = mn$ and

$$E = \frac{mR}{\alpha'} + \frac{n}{R}\big|1 - B_1 mR\big| = \frac{mR}{\alpha'} + \frac{1}{R} . \tag{2.28}$$

The energy is microscopic for $R \sim \sqrt{\alpha'}$ but the string is large in the non compact space: from (2.26) one finds

$$L_1^2 = \frac{\alpha'\big|J_1\big|}{\big|k_1 + mRB_1\big|} = \frac{J_1^2\alpha'}{\big|(mn - mRB_1 J_1)\big|} \tag{2.29}$$

JHEP08(2007)047

which gives $L_1^2 = mn^2\alpha' \gg \alpha'$ for this state. In a coordinate system $\varphi_1' = \varphi_1 + y/mR$ the string state that has energy (2.28) corresponds to a small string with orbital angular momentum in a large orbit.

More generally, for any BPS state one can choose coordinates $\varphi_1' = \varphi_1 - k_1 y/mR$ so that $B_1' = B_1 + k_1/(Rm)$ and the new classical solution is represented by the ansatz $\varphi_1' = \omega_1'\tau$, therefore $k_1' = 0$, with the same $J_1' = J_1$, $m' = m$ and zero momentum along y, i.e. $n' = 0$. It represents a string extended only in the compact dimension y, and looking like a particle on a large cyclotron orbit with radius L_1 in the non-compact space. In this solution, instead of a large size we get a large radius of the orbit. This is in agreement with the periodicity of the spectrum mentioned in section 2.1.1.

Later we will be interested in the case of Dp branes in a background with a RR F_{p+2} flux where the full quantum spectrum is not known and it is not clear whether the spectrum can have any periodicity.

2.3 Supersymmetry

In this section we will prove that the circular strings (2.19) with $B_1 = B_2$ preserve a fraction 1/4 of the 16 supersymmetries of the background (2.1).

As shown in [6], the background with $B_1 = B_2$ preserves 16 supersymmetries satisfying the condition

$$(1 - \Gamma_{1234})\varepsilon = 0\,, \tag{2.30}$$

where $\Gamma_{1234} = \Gamma_1\Gamma_2\Gamma_3\Gamma_4$ and Γ_μ are the ten-dimensional Dirac matrices, $\{\Gamma_\mu,\, \Gamma_\nu\} = 2g_{\mu\nu}$. As we will see below, $\Gamma_{1234} = \gamma_{1234}$ where γ_μ are the Minkowski space Dirac matrices, $\{\gamma_\mu,\, \gamma_\nu\} = 2\eta_{\mu\nu}$.

In type IIA theory, for the circular string (2.19) with winding and momentum we will find in addition two conditions:

$$\gamma_{05}\varepsilon = \mp\varepsilon\,, \qquad \gamma_{05}\gamma_{11}\varepsilon = -\varepsilon\ . \tag{2.31}$$

The minus or plus sign arise for the Right or Left circular string (they are related to the signs in $\kappa^2 = (mR \pm q)$). From these conditions we conclude that the circular string of section 2.2 preserves 1/4 supersymmetries of the background, leaving four unbroken supersymmetries. The reduction of 1/4 is due to the presence of two charges, winding (fundamental string charge) and momentum (the "wave").

Remarkably, the conditions (2.31) are the same conditions that one obtains in the $B_1 = B_2 = 0$ case, so the interaction with the magnetic background does not break any additional supersymmetry. There is only a reduction by a factor 1/2 because the background $B_1 = B_2$ itself preserves 1/2 of the 32 supersymmetries of the type IIA theory.

1. A classical string solution is supersymmetric if there exist covariantly constant (Killing) spinors ε such that [16, 17]

$$\Gamma\varepsilon = \varepsilon\,, \tag{2.32}$$

where Γ is the κ-symmetry matrix [18], which in the type IIA theory is given by

$$\Gamma = \frac{1}{\sqrt{-\det G_{\alpha\beta}}}\dot{X}^\mu X'^\nu \Gamma_{\mu\nu}\gamma_{11}\,, \qquad \Gamma_{\mu\nu} = \frac{1}{2}[\Gamma_\mu,\, \Gamma_\nu]\,, \tag{2.33}$$

and $G_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu}$. We begin by using Cartesian coordinates. The matrices Γ_μ must satisfy the Dirac algebra $\{\Gamma_\mu, \Gamma_\nu\} = 2g_{\mu\nu}$ for the metric (2.1), which in Cartesian coordinates becomes,

$$\begin{aligned} \mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}x_s^2 + \big(1 + B_1^2(x_1^2+x_2^2) + B_2^2(x_3^2+x_4^2)\big)\mathrm{d}x_5^2 + \mathrm{d}x_1^2 + \mathrm{d}x_2^2 + \mathrm{d}x_3^2 + \mathrm{d}x_4^2 \\ + 2B_1x_1\mathrm{d}x_2\mathrm{d}x_5 - 2B_1x_2\mathrm{d}x_1\mathrm{d}x_5 + 2B_2x_3\mathrm{d}x_4\mathrm{d}x_5 - 2B_2x_4\mathrm{d}x_3\mathrm{d}x_5 \,, \end{aligned} \tag{2.34}$$

where $i = 6, \ldots, 9$. The Dirac matrices are given by

$$\begin{aligned} &\Gamma_\mu = \gamma_\mu \,, \quad \mu \neq 5 \,, \\ &\Gamma_5 = \gamma_5 + B_1\big(x_1\gamma_2 - x_2\gamma_1\big) + B_2\big(x_3\gamma_4 - x_4\gamma_3\big) \,. \end{aligned} \tag{2.35}$$

Consider the solution (2.19) $(t \equiv X_0,\ y \equiv X_5)$

$$\begin{aligned} &X_0 = \kappa\tau \,, && X_5 = mR\sigma + q\tau \,, \\ &X_1 = L_1\cos(k_1\sigma + \omega_1\tau) \,, && X_2 = L_1\sin(k_1\sigma + \omega_1\tau) \,, \\ &X_3 = L_2\cos(k_2\sigma + \omega_2\tau) \,, && X_4 = L_2\sin(k_2\sigma + \omega_2\tau) \,. \end{aligned} \tag{2.36}$$

We now consider eq. (2.33). Making use of the results of the previous subsection 2.2, the determinant of the induced metric can be written as

$$\sqrt{-\det G_{\alpha\beta}} = m^2R^2 + \sum_{a=1}^{2} L_a^2\big(k_a + mRB_a\big)^2 \,. \tag{2.37}$$

and

$$\begin{aligned} \dot{X}^\mu X'^\nu \Gamma_{\mu\nu} &= -\kappa k_1 x_2 \Gamma_{01} + \kappa k_1 x_1 \Gamma_{02} - \kappa k_2 x_4 \Gamma_{03} + \kappa k_2 x_3 \Gamma_{04} + \kappa mR\Gamma_{05} \\ &\quad + \big(x_1\Gamma_{25} - x_2\Gamma_{15}\big)\big(\omega_1 mR - k_1 q\big) + \big(x_3\Gamma_{45} - x_4\Gamma_{35}\big)\big(\omega_2 mR - k_2 q\big) \,. \\ &= \kappa mR\gamma_{05} + \sum_{a=1}^{2} x_{2a-1}\Big\{\kappa(k_a + mRB_a)\gamma_{0\,2a} + (\omega_a mR - k_a q)\gamma_{2a\,5}\Big\} \\ &\quad - \sum_{a=1}^{2} x_{2a}\Big\{\kappa(k_a + mRB_a)\gamma_{0\,2a-1} + (\omega_a mR - k_a q)\gamma_{2a-1\,5}\Big\} \,. \end{aligned} \tag{2.38}$$

Hence, the supersymmetry condition (2.33) becomes

$$\left(A\gamma_{05} - \sum_{a=1}^{2} \epsilon^{ij} x_{2a-i}\big\{P_a\gamma_{0\,2a-j} + Q_a\gamma_{2a-j\,5}\big\}\right)\gamma_{11}\varepsilon = \varepsilon \,, \qquad i,j = 0,1 \,, \tag{2.39}$$

where ϵ^{ij} is the complete antisymmetric Levi-Civita symbol, with $\epsilon^{01} = 1$ and

$$A = \frac{\kappa mR}{\sqrt{-\det G_{\alpha\beta}}} \,, \qquad P_a = \frac{\kappa(k_a + mRB_a)}{\sqrt{-\det G_{\alpha\beta}}} \,, \qquad Q_a = \frac{\omega_a mR - k_a q}{\sqrt{-\det G_{\alpha\beta}}} \,. \tag{2.40}$$

If we chose a constant spinor, $\varepsilon = \varepsilon_0$, eq. (2.39) implies

$$A\gamma_{05}\gamma_{11}\varepsilon_0 = \varepsilon_0 \,, \tag{2.41}$$

$$(P_a\gamma_{0b} + Q_a\gamma_{b5})\gamma_{11}\varepsilon_0 = 0 \,, \qquad a = 1,2 \,, \quad b = 1, \ldots, 4 \,. \tag{2.42}$$

JHEP08(2007)047

The first equation will leave half of the supersymmetries of the background if, and only if, $A = \pm 1$. Multiplying the second condition by γ_{b5} from the left we end up with

$$(P_a \gamma_{05} - Q_a)\gamma_{11}\varepsilon_0 = 0\,, \tag{2.43}$$

which requires $P_a = \pm Q_a$. The classical string solution must therefore satisfy the conditions

$$\begin{aligned} \kappa m R &= m^2 R^2 + \sum_{a=1}^{2} L_a^2 \big(k_a + B_a m R\big)^2\,, \\ \kappa(k_a + mRB_a) &= \pm(\omega_a m R - k_a q)\,, \qquad a = 1,2\,. \end{aligned} \tag{2.44}$$

From the constraints (2.21) and (2.24) one can easily see that these conditions are indeed satisfied. Thus we have two conditions on the Killing spinors:

$$\gamma_{05}\varepsilon_0 = \mp\varepsilon_0\,, \qquad \gamma_{05}\gamma_{11}\varepsilon_0 = -\varepsilon_0\,, \tag{2.45}$$

as anticipated above, showing that the classical string solution preserves 1/4 of the 16 supersymmetries of the $B_1 = B_2$ background. Note that conditions (2.44) are also satisfied if the string rotates only on one plane, i.e. $L_2 = 0$, $\omega_2 = k_2 = 0$.
2. It is instructive to repeat the previous derivation in polar coordinates. For simplicity, here we will consider a string rotating only in the plane 12,

$$t = \kappa\tau\,, \qquad r_1 = r_0\,, \qquad \varphi_1 = \omega\tau + k\sigma\,, \qquad y = q\tau + mR\sigma\,, \tag{2.46}$$

and the remaining coordinates equal to zero.

The relevant Γ matrices are found to be

$$\Gamma_0 = \gamma_0\,, \qquad \Gamma_\varphi = r_0\gamma_1\,, \qquad \Gamma_y = \gamma_5 + r_0 B\gamma_1\,, \tag{2.47}$$

in fact $\{\Gamma_\varphi, \Gamma_\varphi\} = 2r_0^2$, $\{\Gamma_\varphi, \Gamma_y\} = 2r_0^2 B$, $\{\Gamma_y, \Gamma_y\} = 2(1 + r_0^2 B^2)$.

We get the condition

$$\big(r_0 P_1 \gamma_0\gamma_1 + A\gamma_0\gamma_5 + r_0 Q_1 \gamma_1\gamma_5\big)\gamma_{11}\varepsilon = \varepsilon\,, \tag{2.48}$$

where P_1, Q_1, A were given in (2.40), (2.37), now with $(L_1, L_2) = (r_0, 0)$. This leads to the same conditions (2.45) as in the previous derivation.

3. Magnetic field from Kaluza-Klein reduction from $B_{\mu\nu}$

3.1 The quantum string spectrum

The model is obtained by a T-dual transformation in the y direction from the previous model (2.1). We recall the standard rules of T-duality [19]:

$$g'_{yy} = g_{yy}^{-1}\,, \qquad e^{2\phi'} = \frac{e^{2\phi}}{g_{yy}}\,, \qquad B'_{y\mu} = \frac{g_{y\mu}}{g_{yy}}\,, \qquad g'_{y\mu} = \frac{B_{y\mu}}{g_{yy}}\,. \tag{3.1}$$

We get (dropping "primes")

$$
\begin{aligned}
ds^2 &= -dt^2 + dx_s^2 + dr_1^2 + dr_2^2 + r_1^2 d\varphi_1^2 + r_2^2 d\varphi_2^2 + \Lambda^{-1}\left(dy^2 - \left(B_1 r_1^2 d\varphi_1 + B_2 r_2^2 d\varphi_2\right)^2\right), \\
e^{2(\phi-\phi_0)} &= \Lambda^{-1}, \qquad \mathrm{B}_2 = \Lambda^{-1}(B_1 r_1^2 d\varphi_1 + B_2 r_2^2 d\varphi_2) \wedge dy\,, \\
\Lambda &= 1 + B_1^2 r_1^2 + B_2^2 r_2^2\,.
\end{aligned}
\tag{3.2}
$$

It represents an exact solution of string theory to all α' orders (being related by T-duality to a flat spacetime).

The string spectrum is obtained from the spectrum of the previous model by exchanging m and n and $R \to \alpha'/R$. Thus

$$
\begin{aligned}
\alpha' M^2 &= 2(N_L + N_R) + \alpha'\left(\frac{mR}{\alpha'} - B_1 J_1 - B_2 J_2\right)^2 + \frac{n^2}{R^2} \\
&\quad - 2\alpha' B_1 \frac{n}{R}(J_{1R} - J_{1L}) - 2\alpha' B_2 \frac{n}{R}(J_{2R} - J_{2L})\,, \\
N_R - N_L &= mn\,.
\end{aligned}
\tag{3.3}
$$

Now the spectrum is periodic in the parameters $\gamma_{1,2} \equiv \alpha' B_{1,2}\frac{n}{R}$ with period 1.

The state dual to (2.6) has zero winding $m = 0$ and $n = 1$ and becomes tachyonic for $B_1 - B_2 > 1/(2R)$. Note that this is a Kaluza-Klein state of the supergravity multiplet. This tachyon was studied in [4–6].

Consider the supersymmetric model $B_1 = B_2 \equiv B$. The BPS states have similar quantum numbers as in the previous T-dual case:

$$
N_L = 0\,, \qquad N_R = mn\,, \qquad (S_{1L}, S_{2L}) = (-1, 0) \ \text{ or } \ (0, -1)\,, \tag{3.4}
$$

and mass given by (cf. eq. (2.16))

$$
M_{\mathrm{BPS}} = \left|\frac{n}{R}\right| + \left|\frac{mR}{\alpha'} - B(J_1 + J_2)\right|\,. \tag{3.5}
$$

B is now restricted to be in the interval $0 < \alpha' B\frac{n}{R} < 1$, outside which the spectrum is repeated periodically. For the states with maximum spin $S_{1R} + S_{2R} = N_R + 1$ and $l_{1,2R} = 0$ we get

$$
M_{\mathrm{BPS}} = \left|\frac{n}{R}\right| + \left|\frac{mR}{\alpha'} - Bnm\right|\,, \tag{3.6}
$$

The minimum mass is achieved for $\alpha'\frac{n}{R}B \to 1$, where the mass becomes

$$
M_{\mathrm{BPS}} \to \left|\frac{n}{R}\right|\,. \tag{3.7}
$$

The macroscopic strings becoming light are now strings with large m and small n. We will see that their size is much greater than the string length $\sqrt{\alpha'}$.

3.2 Classical solution for the BPS string using Polyakov formalism

We consider a string rotating on one plane 12 only, and set $r_2 = \varphi_2 = 0$. The Polyakov action becomes

$$S = \frac{1}{2\pi}\int d\tau d\sigma \; L\,, \tag{3.8}$$

with

$$L = \frac{1}{2\alpha'}\left(-\dot{t}^2+\dot{r}_1^2-{r'_1}^2+\frac{1}{1+B_1^2r_1^2}\left(r_1^2(\dot{\varphi}_1^2-{\varphi'_1}^2)+\dot{y}^2-{y'}^2\right)+\frac{2B_1r_1^2}{1+B_1^2r_1^2}\left(\dot{\varphi}_1y'-\varphi'_1\dot{y}\right)\right)\,, \tag{3.9}$$

where $\dot{x}\equiv\frac{\partial x}{\partial\tau}$ and $x'\equiv\frac{\partial x}{\partial\sigma}$.

The ansatz for the circular string is:

$$\dot{t}=\kappa\,,\quad \dot{r}_1=r'_1=0\;\rightarrow\; r_1=r_0\,,\quad \dot{\varphi}_1=\omega\,,\quad \varphi'_1=k\,,\quad \dot{y}=q\,,\quad y'=mR\,. \tag{3.10}$$

The equation of motion for $\varphi_1,\ y$ are automatically satisfied and the equation for r_1 gives

$$\left.\frac{\partial L}{\partial r_1^2}\right|_{r_1=r_0}=0\quad\rightarrow\quad(\omega+B_1mR)^2=(k+B_1q)^2\,. \tag{3.11}$$

The conserved (quantized) momenta are

$$\begin{aligned}\alpha' J_1 &= \frac{\partial L}{\partial\omega}=\frac{r_0^2}{1+B_1^2r_0^2}(\omega+B_1mR)\,,\\ \alpha'\frac{n}{R}&=\frac{\partial L}{\partial q}=q-\frac{B_1r_0^2}{1+B_1^2r_0^2}(k+B_1q)\,,\end{aligned} \tag{3.12}$$

giving

$$\begin{aligned}r_0^2\omega&=\alpha' J_1(1+B_1^2r_0^2)-B_1r_0^2mR\,,\\ q&=\alpha'\frac{n}{R}(1+B_1^2r_0^2)+r_0^2B_1k\,.\end{aligned} \tag{3.13}$$

From the Virasoro constraint, we have $r_0^2\omega k+qmR=0$, which becomes

$$J_1k+nm=0\,, \tag{3.14}$$

we get

$$\begin{aligned}q^2+r_0^2\omega^2&=m^2R^2\left(1+\frac{n^2}{R^2}\frac{r_0^2}{J_1^2}\right)D^2\,,\qquad D^2\equiv\frac{1}{r_0^2}\left[B_1\left(1-\alpha'\frac{B_1J_1}{mR}\right)r_0^2-\alpha'\frac{J_1}{mR}\right]^2\,,\\ m^2R^2+r_0^2k^2&=m^2R^2\left(1+\frac{n^2}{R^2}\frac{r_0^2}{J_1^2}\right)\,.\end{aligned} \tag{3.15}$$

The remaining Virasoro constraint gives the energy, which takes the form

$$E^2=\frac{\kappa^2}{\alpha'^2}=\frac{1}{1+B_1^2r_0^2}\frac{m^2R^2}{\alpha'^2}\left(1+\frac{n^2}{R^2}\frac{r_0^2}{J_1^2}\right)(1+D^2)\,. \tag{3.16}$$

From the equations (3.11), (3.12) we get

$$\frac{J_1^2}{r_0^4} = \frac{1}{\alpha'^2}\left(k - \alpha' B_1 \frac{n}{R}\right)^2 = \frac{n^2}{R^2}\left(\frac{mR}{\alpha' J_1} - B_1\right)^2 , \tag{3.17}$$

which gives r_0 in terms of the (quantized) quantum numbers and B_1 (compare with (2.29)):

$$r_0^2 = \frac{\alpha' |J_1|}{|k_1 - \alpha' \frac{n}{R} B_1|} = \frac{J_1^2}{|\frac{n}{R}(\frac{mR}{\alpha'} - B_1 J_1)|} \tag{3.18}$$

Substituting this value we get

$$E = \left|\frac{n}{R}\right| + \left|\frac{mR}{\alpha'} - B_1 J_1\right| . \tag{3.19}$$

This reproduces exactly the energy formula (3.5) of the quantum string spectrum for $J_2 = 0$.

3.3 Classical solution for the BPS string using Nambu-Goto formalism

Since we will be later interested in studying Dp branes, whose dynamics is governed by the Dirac-Born-Infeld (DBI) action, it is useful to reproduce the previous result in the Nambu-Goto formalism. We will see that the equations that we find for the Dp brane analog of the rotating circular BPS string are a simple generalization of the treatment described below.

The Nambu-Goto Lagrangian is given by

$$L = \frac{1}{\alpha'}\sqrt{-\det G_{\alpha\beta}} + \frac{1}{\alpha'} B_{\mu\nu} \epsilon^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu , \qquad G_{\alpha\beta} = g_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu , \tag{3.20}$$

where $\alpha, \beta = \sigma, \tau$.

We will work in the gauge $G_{\sigma\tau} = 0$. By taking the same anstatz as in section 3.2 with $r_1 = r_0$ and $r_2 = 0$ (i.e. the string rotating in one plane one only) we get

$$L = \frac{1}{\alpha'\Lambda}\sqrt{(\kappa^2\Lambda - q^2 - r_0^2\omega^2)(r_0^2 k^2 + m^2 R^2)} + \frac{B_1}{\alpha'\Lambda} r_0^2 (\omega m R - qk) , \tag{3.21}$$

with $\Lambda = 1 + B_1^2 r_0^2$. The energy, angular momentum J_1 in the plane r_1, φ_1 and the linear momentum in y are obtained by

$$\begin{aligned}
E &= \frac{\partial L}{\partial \kappa} = \frac{1}{\alpha'}\sqrt{\frac{r_0^2 k^2 + m^2 R^2}{\Lambda - U}} , \\
J_1 &= \frac{\partial L}{\partial \omega} = -\frac{r_0^2 \omega}{\alpha' \kappa \Lambda}\sqrt{\frac{r_0^2 k^2 + m^2 R^2}{\Lambda - U}} + \frac{B_1 r_0^2 m R}{\alpha' \Lambda} , \\
\frac{n}{R} &= \frac{\partial L}{\partial q} = -\frac{q}{\alpha' \kappa \Lambda}\sqrt{\frac{r_0^2 k^2 + m^2 R^2}{\Lambda - U}} - \frac{B_1 r_0^2 k}{\alpha' \Lambda} ,
\end{aligned} \tag{3.22}$$

where

$$U = \frac{q^2 + r_0^2 \omega^2}{\kappa^2} . \tag{3.23}$$

The constraint equation $G_{\sigma\tau} = 0$ becomes

$$kJ_1 + nm = 0 \ . \tag{3.24}$$

The equations (3.22) can be combined giving

$$\frac{U}{\Lambda - U} = \frac{1}{r_0^2}\left[B_1 r_0^2\left(1 - \frac{\alpha' B_1 J_1}{mR}\right) - \frac{\alpha' J_1}{mR}\right]^2 \equiv D^2 \,, \tag{3.25}$$

or

$$U = \Lambda \frac{D^2}{1 + D^2} \ . \tag{3.26}$$

Note that D^2 is the same quantity seen in the previous subsection eq. (3.15). The energy square of the state becomes the same expression eq. (3.16), which we now write exhibiting the explicit r_0 dependence.

$$E^2 = \frac{m^2R^2}{\alpha'^2}\frac{1 + \frac{n^2}{R^2}\frac{r_0^2}{J_1^2}}{1 + B_1^2 r_0^2}\left(1 + \left[\frac{B_1}{r_0^2}(1 - \frac{\alpha' B_1 J_1}{mR}) - \frac{\alpha' J_1}{mR r_0^4}\right]^2\right) \ . \tag{3.27}$$

The Hamiltonian that arises after the gauge choice $X_0 = \kappa\tau$ is (after substituting for q, ω their expressions in terms of $\frac{n}{R}, J_1$)

$$H\big(n/R, J_1, r_0\big) = \frac{n}{R}\frac{\partial y}{\partial \tau} + J_1 \frac{\partial \varphi_1}{\partial \tau} - L = \frac{n}{R} q + J_1 \omega - L \ . \tag{3.28}$$

Due to τ-scaling invariance of the Lagrangian we have $H = -E$. Therefore the equation for r_0 is $\frac{\partial E}{\partial r_0} = 0$ and this is seen to give the same eq. (3.17) as in the previous subsection. Therefore

$$E = \left|\frac{n}{R}\right| + \left|\frac{mR}{\alpha'} - B_1 J_1\right| \ . \tag{3.29}$$

In this way we recover the result (3.19) found by using the Polyakov formalism.

The classical description reproduces the energy of the quantum string spectrum for large quantum numbers also in non-supersymmetric configurations. As an example, in the appendix we compute the classical energy of a rotating folded string.

4. Dp-branes interacting with a magnetic RR flux

We consider the S-dual background to (3.2), which is given by

$$\begin{aligned} ds^2 &= \Lambda^{\frac{1}{2}}\big(- dt^2 + dx_s^2 + dr_1^2 + dr_2^2 + r_1^2 d\varphi_1^2 + r_2^2 d\varphi_2^2\big) \,, \\ &\quad + \Lambda^{-\frac{1}{2}}\big(dy^2 - (B_1 r_1^2 d\varphi_1 + B_2 r_2^2 d\varphi_2)^2\big) \,, \\ e^{2(\phi - \phi_0)} &= \Lambda \,, \\ A_2 &= e^{-\phi_0}\Lambda^{-1}(B_1 r_1^2 d\varphi_1 + B_2 r_2^2 d\varphi_2) \wedge dy \ . \end{aligned} \tag{4.1}$$

where $\Lambda = 1 + B_1^2 r_1^2 + B_2^2 r_2^2$. This represents a solution to the classical string equations to the leading order in α' . This background contains a flux which couples to a D string.

In order to obtain magnetic flux backgrounds for the Dp brane, we perform T-duality transformations on x_s coordinates. Using the usual rules (given in [20]) one finds

$$\begin{aligned} ds^2 &= \Lambda^{\frac{1}{2}}\big(- dt^2 + dx_s^2 + dr_1^2 + dr_2^2 + r_1^2 d\varphi_1^2 + r_2^2 d\varphi_2^2\big) \\ &\quad + \Lambda^{-\frac{1}{2}}\big(dy_1^2 + \ldots + dy_p^2 - (B_1 r_1^2 d\varphi_1 + B_2 r_2^2 d\varphi_2)^2\big)\,, \\ e^{2(\phi-\phi_0)} &= \Lambda^{3/2-p/2}\,, \\ A_{p+1} &= e^{-\phi_0}\Lambda^{-1}(B_1 r_1^2 d\varphi_1 + B_2 r_2^2 d\varphi_2) \wedge dy_1 \wedge dy_2 \wedge \ldots \wedge dy_p\ . \end{aligned} \tag{4.2}$$

Here $s = p+5, \ldots, 9$.

For simplicity, here we consider a Dp-brane with $p \geq 1$ which rotates only on the 12 plane, lying at $r_2 = 0$. In this case the dependence on B_2 disappears. The projection of this Dp brane on the plane 12 describes a circle with radius $r_1 = r_0$. The Dp brane also moves and winds on a $p-$dimensional torus in the compact space. The compact coordinates y_i have periodicity $y_i \sim y_i + 2\pi R_i$, $i = 1, \cdots, p$. It is convenient to formally describe the trajectory in the non-compact space r_1, φ_1 in terms of another circular coordinate $y_0 = r_0\varphi_1$ such that $y_0 \sim y_0 + 2\pi r_0$.

The Dp brane action is given by

$$S = \frac{1}{(2\pi)^p}\int d\tau \prod_{l=1}^{p}\int_0^{2\pi} d\sigma_l\ L\,, \tag{4.3}$$

where for this ($r_2 = 0$) Dp brane the Lagrangian L becomes

$$L = \mu_p e^{-(\phi-\phi_0)}\sqrt{-\det G_{\alpha\beta}} + \frac{\mu B_1}{\Lambda} r_1\ \det M\,, \tag{4.4}$$

where

$$\begin{aligned} \mu_p &= \frac{1}{g_s l_s^{p+1}}\,, \\ g_s &= e^{\phi_0}\,, \\ l_s &= \sqrt{\alpha'}\ . \end{aligned} \tag{4.5}$$

Here $\alpha, \beta = 0, 1, \cdots, p$ and

$$G_{\alpha\beta} = g_{\mu\nu}\frac{dX^\mu}{ds_\alpha}\frac{dX^\nu}{ds_\beta}\,, \tag{4.6}$$

with $ds_\alpha = (d\tau, d\sigma_1, \cdots, d\sigma_p)$ and

$$M_{\alpha\beta} \equiv \frac{dy_\beta}{ds_\alpha}\,, \tag{4.7}$$

with $y_\beta = (y_0, y_1, \cdots, y_p)$. Note the dimension $[B_1] = [1/R]$.

We take the ansatz:

$$X^0 = \kappa\tau\,, \qquad r_1 = r_0\,, \qquad r_2 = 0\,, \qquad \varphi_1 = \omega\tau + k_i\sigma_i\,, \qquad y_i = q_i\tau + \tilde{m}_{ij}\sigma_j\ . \tag{4.8}$$

Using our compact notation, we write $y_0 = q_0\tau + \tilde{m}_{0j}\sigma_j$ with $q_0 = r_0\omega$ and $\tilde{m}_{0j} \equiv r_0 k_j$. Note that $q_\alpha \equiv \dot{y}_\alpha$ and the momenta are $p_{y_\alpha} = \frac{\partial L}{\partial \dot{y}_\alpha}$.

The constants of motions are:

- k_i;
- the compact windings m_{ij}: $\tilde{m}_{ij} \equiv R_i m_{ij}$ (for $y_i \sim y_i + 2\pi R_i$);
- the momenta in the compactified directions $p_{y_i} = \frac{n_i}{R_i} = \frac{\partial L}{\partial q_i}$;
- the angular momentum $J_1 = \frac{\partial L}{\partial \omega}$ corresponding to the "momentum" $p_{y_0} = \frac{J_1}{r_0} = \frac{\partial L}{\partial q_0}$.

Note that $k_i,\ m_{ij},\ J_1,\ n_i$ are integer numbers.

We take the gauge

$$G_{0j} = g_{\mu\nu} \frac{dX^\mu}{d\tau} \frac{dX^\mu}{d\sigma_j} = 0\,, \qquad j = 1, \cdots, p\,. \tag{4.9}$$

This can be rewritten as $\sum_{\alpha=0}^{p} q_\alpha \tilde{m}_{\alpha j} = 0$ for each $j = 1, \cdots, p$. Since $\frac{\partial \det(M)}{\partial q_\alpha} \tilde{m}_{\alpha j} = 0$ for $j = 1, \cdots, p$ and $\frac{\partial \sqrt{-\det G}}{\partial q_\alpha} \sim q_\alpha$, this implies

$$\sum_{\alpha=0}^{p} \frac{\partial L}{\partial q_\alpha} \tilde{m}_{\alpha j} = \sum_{\alpha=0}^{p} p_{y_\alpha} \tilde{m}_{\alpha j} = J_1 k_j + \sum_{i=1}^{p} n_i m_{ij} = 0\,, \qquad j = 1, \cdots, p\,. \tag{4.10}$$

Another consequence is that the product of the matrix M times its transpose, i.e. $M \cdot M^T$, is block-diagonal and one gets $\det(M) = q\sqrt{\det(G_{ij})}$ with $q \equiv \sqrt{\sum_{\alpha=0}^{p} q_\alpha^2}$. Also, note that $G_{00} = \kappa^2 \Lambda - q^2$.

Therefore we can rewrite the Lagrangian in this gauge as:

$$L = \frac{\mu_p}{\Lambda} \sqrt{(\kappa^2 \Lambda - q^2) \det(G_{ij})} + \frac{\mu_p B_1 r_0}{\Lambda} q \sqrt{\det(G_{ij})}\,. \tag{4.11}$$

Consider now, for fixed G_{ij}, the vector $\vec{q}$ whose components are the dynamical variables $q_\alpha = \dot{y}_\alpha$. We note that L is invariant under rotations of $\vec{q}$. Therefore we can take a frame where $\vec{q} = (q, 0, \cdots, 0)$ getting

$$p_y \equiv \sqrt{\sum_{\alpha=0}^{p} p_{y_\alpha}^2} = \frac{\partial L}{\partial q} = \frac{-q}{\sqrt{\kappa^2 \Lambda - q^2}} \frac{\mu_p}{\Lambda} \sqrt{\det(G_{ij})} + \frac{\mu_p B_1 r_0}{\Lambda} \sqrt{\det(G_{ij})}\,, \tag{4.12}$$

from which it follows that

$$\frac{q^2}{\kappa^2} = \Lambda \frac{D^2}{1 + D^2}\ , \qquad D^2 \equiv \left(\frac{\Lambda}{\mu_p} \frac{\sqrt{(J_1/r_0)^2 + \sum_{i=1}^{p} (n_i/R_i)^2}}{\sqrt{\det(G_{ij})}} - B_1 r_0 \right)^2 . \tag{4.13}$$

By using the relations (4.10) one verifies the following identity:

$$\frac{\sqrt{(J_1/r_0)^2 + (n/R)^2}}{\sqrt{\det(G_{ij})}} = \frac{J_1/r_0}{\tilde{m}}\ , \qquad \tilde{m} \equiv \det(\tilde{m}_{ij})\,, \qquad \frac{n}{R} \equiv \sqrt{\sum_{i=1}^{p} \frac{n_i^2}{R_i^2}}\ . \tag{4.14}$$

We fix $r_1 = r_0$ by requiring $\frac{\partial H}{\partial r_0} = 0$ where H is the Hamiltonian. This Hamiltonian describing the dynamics of the space coordinates at fixed κ is obtained by substituting for q_i, ω their expressions in terms of $\frac{n_i}{R_i}, J_1$ in the general formula

$$H\big(\frac{n_i}{R_i}, J_1, r_0\big) = \sum_{i=1}^{p} \frac{n_i}{R_i}\frac{\partial y_i}{\partial \tau} + J_1 \frac{\partial \varphi}{\partial \tau} - L = \sum_i \frac{n_i}{R_i} q_i + J_1 \omega - L \ . \tag{4.15}$$

Due to τ-scaling invariance of the Lagrangian we have $H = -E$, where the energy of the state is

$$\begin{aligned} E \equiv \frac{\partial L}{\partial \kappa} = \mu_p \sqrt{\frac{\det(G_{ij})}{\Lambda - q^2/\kappa^2}} &= \mu_p \sqrt{\frac{\det(G_{ij})(1+D^2)}{\Lambda}} \\ &= \mu_p \tilde{m} \sqrt{\frac{1 + \frac{n^2}{R^2}\frac{r_0^2}{J_1^2}}{1 + B_1^2 r_0^2}\left(1 + \left[\frac{B_1}{r_0^2}(1 - \frac{B_1 J_1}{\mu_p \tilde{m}}) - \frac{J_1}{\mu_p \tilde{m} r_0^4}\right]^2\right)} \ . \end{aligned} \tag{4.16}$$

In the last step we have used the definitions and the identities (4.13) and (4.14).

From now on, the calculation follows identical steps as in section 3.3. We find r_0 by the equation $\frac{\partial E^2}{\partial r_0^2} = 0$. This gives again

$$r_0^2 = \frac{J_1^2}{\frac{n}{R}|\tilde{m}\mu_p - B_1 J_1|} \ . \tag{4.17}$$

Substituting this value into the above expression for E we get

$$E = \frac{n}{R} + \left|\tilde{m}\mu_p - B_1 J_1\right| = \sqrt{\sum_{i=1}^{p} \frac{n_i^2}{R_i^2}} + \left|R_1 \cdot R_2 \cdots R_p \ \det(m_{ij})\mu_p - B_1 J_1\right| . \tag{4.18}$$

In terms of the tension of the Dp brane $\tau_p = \mu_p/(2\pi)^p$ and the volume of the torus T^p this becomes

$$E = \sqrt{\sum_{i=1}^{p} \frac{n_i^2}{R_i^2}} + \left|\tau_p \, \mathrm{Vol}(T^p)\, m - B_1 J_1\right| , \tag{4.19}$$

where

$$m \equiv \det(m_{ij}) = \frac{1}{\mathrm{Vol}(T^p)} \int dy_1 \wedge \cdots \wedge dy_p \ . \tag{4.20}$$

is the winding number of the Dp brane around the T^p-torus. Note that the term $\tau_p \times \mathrm{Vol}(T^p) \times m$ is the expected contribution to the energy of the form tension $\times$ volume $\times$ winding. This term is $O(1/g_s)$.

Thus we find that the energy has the same form as the energy (3.19) of the circular BPS string (in the particular case $J_2 = 0$). This Dp brane solution is also supersymmetric as it is related by dualities to the circular BPS string. In the particular $p = 2$ and $B_1 = B_2 = 0$ case, it agrees with the energy formula for the analogous membrane BPS solution found in [21, 22] (see eq. (2.42) in [22]).

JHEP08(2007)047

Let us now consider some physical implications of the energy formula. We take $R_i \sim l_s$. In the absence of magnetic fields and assuming $g_s \ll 1$ the energy of the brane is essentially given by the winding contribution

$$E \sim \frac{1}{g_s l_s} m \gg \frac{1}{l_s} \ . \tag{4.21}$$

Similarly to the case of the string, we find that there are Dp branes which become light when the magnetic flux gets to some value, $B_1 J_1 \sim \tau_p \mathrm{Vol}(T^p) m$. These are states with large m, and with n_i of order 1. The energy becomes

$$E \sim \sqrt{\sum_{i=1}^{p} \frac{n_i^2}{R_i^2}} \ll \frac{1}{g_s l_s} m \ . \tag{4.22}$$

Strikingly, these Dp branes become macroscopic since r_0 (and in fact the proper distance) goes to infinity when B_1 approaches $\tau_p \mathrm{Vol}(T^p) m / J_1$ (see eq. (4.17)). In the absence of magnetic fields they have size $r_0 \sim \frac{J_1}{\sqrt{m}} l_s \sqrt{g_s}$, which is typically small in the perturbative regime, but it may be large by a suitable choice of quantum numbers (satisfying of course the constraint (4.10)).

In the case of the model (2.1), the physical string spectrum is periodic in the magnetic field parameters. A very interesting open question is whether the full quantum spectrum of Dp brane states in the background (4.2) could also have some analogous periodicity.

5. Discussion

To summarize, we have computed the energy of Dp branes in the presence of magnetic RR flux backgrounds and identified a family of BPS rotating Dp brane solutions which are invariant under four supersymmetry transformations. There are some potentially interesting applications. Since the solutions are BPS, the mass formula should be protected from quantum string theory corrections and therefore it should be possible to extrapolate it to strong coupling, where the gravitational field of the brane becomes important and these branes could become black holes, analogous to the black holes of [23], but moving in magnetic fields. Also, it may be possible to construct the Dp brane supergravity solution with the addition of the magnetic RR F_{p+2} flux by starting with a Dp' brane background, adding magnetic parameters as in section 2 and perform several dualities, providing a model for AdS/CFT correspondence which might exhibit some interesting effects.

We have seen that there are macroscopic string and Dp brane states which become light for some values of the magnetic field parameters B_1, B_2. In general, the presence of many light classical macroscopic states in a non-supersymmetric background could be a sign of potential instabilities. In the non-supersymmetric case, the quantum spectrum is known [3, 4] to contain tachyons in some range of the parameters B_1, B_2, R (see section 2). Such instabilities in principle can arise both from string modes or from supergravity modes. In the first case, like the model of section 2.1, where tachyons arise in the winding sector, the supergravity background is classically stable, but the string theory is unstable in some range of the parameters. In the second case, like the model of section 3.1, the supergravity background itself is classically unstable (which allows a study of tachyon instabilities in a field theory setting [5]).

From the quantum spectrum (2.3) (see also appendix A), it can be seen that, when $B_1 \neq B_2$, even states with $N_L = 0$, $N_R = mn$ can become tachyonic (when $B_1 = B_2$ the mass squared of these states is manifestly positive definite). The energy of the corresponding classical solutions never becomes imaginary because, from the Virasoro constraint, E^2 is proportional to $(\dot{X}^i \dot{X}^j + X^{i\prime} X^{j\prime}) g_{ij} \geq 0$ where g_{ij} is the spatial part of the metric. The classical string may become light, but not tachyonic.

Since the presence of fluxes in string-theory backgrounds are important for moduli stabilization, a very interesting question is whether instabilities could also arise for non-supersymmetric flux compactification models in some range of the parameters. If this is the case, this effect could constraint the number of stable vacua. A study in this direction was done in [24], looking for instabilities of the supergravity background.

The reduction of the energy of a state originates due to the standard gyromagnetic interaction. This effect is universal and it is present for any quantum state with spin that moves in a magnetic field. In string theory, the effect can be stronger due to the existence of states with arbitrarily large values of the spin, for which the negative gyromagnetic coupling can be important even for weak magnetic fields (for example, there are magnetic string models which become unstable for infinitesimal values of the magnetic field, see section 6 in [3]). For strong magnetic fields, one needs to take into account $O(B^2)$ effects where gravity gets into the game. Finding the full quantum spectrum in this case is in general highly complicated, but we have seen that the energy of quantum states with large spin can be obtained with a good accuracy by studying the classical dynamics. In non-supersymmetric backgrounds, this may signal potential instabilities by the presence of light states with large spin.

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A. Quantum spectrum

Let us review the main points of the string quantization in the background (2.1). We refer to [3] for more details. The coordinate y satisfies the free equation $\partial_+ \partial_- y = 0$. Write $y = q\tau + mR\sigma + y'$, where y' is single-valued and define $\gamma_{1,2} = B_{1,2} mR$, taking $0 \leq \gamma_i < 1$. Introducing complex coordinates in the planes $1, 2$: $Z_{1,2} = X_{1,2} + iY_{1,2} = r_{1,2} e^{i\varphi'_{1,2}}$ with

JHEP08(2007)047

$\varphi'_{1,2} = \varphi_{1,2} + B_{1,2}y$, one gets $Z_i = Z_{iR}(\tau+\sigma) + Z_{iL}(\tau-\sigma)$ where

$$\sqrt{\frac{2}{\alpha'}} Z_{iR}(s) = i a_{i0} e^{i\gamma_i s} + i \sum_{k=1}^{\infty} a_{i,k} e^{i(k+\gamma_i)s} + a_{i,-k} e^{i(-k+\gamma_i)s} \,. \tag{A.1}$$

$Z_{iL}(s)$ has the same expression with $a_{ik} \to \tilde{a}_{ik}$ and $k+\gamma_i \to k-\gamma_i$, and similarly $Z_i^* = X_i - iY_i$ in terms of $a^*_{i,k}$ (R part) and $\tilde{a}^*_{i,k}$ (L part). Note that $Z_{iR,L}(\sigma = 2\pi) = e^{i2\pi\gamma_i} Z_{iR,L}(\sigma = 0)$.

One then introduces annihilation and creation operators by $b_{i0} \equiv \sqrt{\frac{\gamma_i}{2}} a_{i0}\,,\ b^\dagger_{i0} \equiv \sqrt{\frac{\gamma_i}{2}} a^*_{i0}$ and

$$b_{ik-} \equiv \sqrt{\frac{k+\gamma_i}{2}} a_{i,k}\,,\ b^\dagger_{ik-} \equiv \sqrt{\frac{k+\gamma_i}{2}} a^*_{i,k}\,,\ b_{ik+} \equiv \sqrt{\frac{k-\gamma_i}{2}} a^*_{i,-k}\,,\ b^\dagger_{ik+} \equiv \sqrt{\frac{k-\gamma_i}{2}} a_{i,-k}\,,$$

such that $[b_{ik\pm}, b^\dagger_{jk'\pm}] = \delta_{ij}\delta_{kk'}$ and similarly for the Left part. A similar construction holds for the fermionic coordinates. Here we will consider as an example the NS sector. In this case, the integer k is replaced by a half-integer number and therefore there is no fermionic zero mode.

The angular momentum in the plane i is $J_i = J_{iR} + J_{iL}$ with

$$\begin{aligned}
J_{iR} &= -\frac{1}{4}\sum_k (k+\gamma_i)(a^*_{ik}a_{ik} + a_{ik}a^*_{ik}) + J^\psi_{iR} \\
&= -b^\dagger_{i0}b_{i0} - \frac{1}{2} + \sum_{k\geq 1}(b^\dagger_{ik+}b_{ik+} - b^\dagger_{ik-}b_{ik-}) + J^\psi_{iR}\,, \\
J_{iL} &= -\frac{1}{4}\sum_k (k-\gamma_i)(a^*_{ik}a_{ik} + a_{ik}a^*_{ik}) + J^\psi_{iL} \\
&= \tilde{b}^\dagger_{i0}\tilde{b}_{i0} + \frac{1}{2} + \sum_{k\geq 1}(\tilde{b}^\dagger_{ik+}\tilde{b}_{ik+} - \tilde{b}^\dagger_{ik-}\tilde{b}_{ik-}) + J^\psi_{iL}\,,
\end{aligned}$$

where $J^\psi_{iR,L}$ are the contributions of the fermionic coordinates in the plane i. Note that there is no fermionic analog of $b^\dagger_{i0}b_{i0}+\frac{1}{2}$. The momentum in the y direction is $\frac{n}{R} = q + B_1J_1 + B_2J_2$.

One sees that

$$\begin{aligned}
&\sum_i\sum_k \left(\frac{k+\gamma_i}{2}\right)^2 (a^*_{ik}a_{ik} + a_{ik}a^*_{ik}) + 2N'_R = 2N_R - \sum_i 2\gamma_i J_{iR}\,, \\
&\sum_i\sum_k \left(\frac{k-\gamma_i}{2}\right)^2 (\tilde{a}^*_{ik}\tilde{a}_{ik} + \tilde{a}_{ik}\tilde{a}^*_{ik}) + 2N'_L = 2N_L + \sum_i 2\gamma_i J_{iL}\,,
\end{aligned}$$

where $N'_{R,L}$ represent the contributions of the other coordinates and of the NS fermions before normal ordering, and $N_{R,L}$ are the usual flat spacetime number operators including the contribution of every coordinate and of the NS fermions: $N_{R,L} =: N_{R,L} : -1/2$. The GSO projection implies $N_{R,L} \geq 0$. From this we get

$$\alpha' M^2 = \frac{(mR)^2}{\alpha'} + \alpha'\big(\frac{n}{R} - B_1J_1 - B_2J_2\big)^2 + 2(N_R + N_L) - \sum_i 2\gamma_i(J_{iR} - J_{iL})\,, \tag{A.2}$$

where $J_{iR,L} \equiv S_{iR,L} \mp (l_{iR,L} + 1/2)$ and $l_{iR} = b^\dagger_{i0} b_{i0}$, $l_{iL} = \tilde b^\dagger_{i0} \tilde b_{i0}$, representing the contributions of the Landau Levels in the plane i. Note the bounds

$$l_{iR,L} \geq 0\,, \qquad |S_{1R,L} \pm S_{2R,L}| \leq N_{R,L} + 1\ . \tag{A.3}$$

The bound for $S_{iR,L}$ can be saturated if in the plane 1 or 2 the fermionic modes $1/2+$ or $1/2-$ are excited (and no other fermionic modes) plus possibly the modes $1+$ or $1-$ (and no other bosonic modes). From (A.3) one can see that

$$\begin{aligned}\alpha' M^2 \geq \frac{(mR)^2}{\alpha'} + \alpha'\left(\frac{n}{R} - B_1 J_1 - B_2 J_2\right)^2 + 2(N_R + N_L)\left(1 - \sum_i \frac{\gamma_i}{2}\right) \\ -(\gamma_1 - \gamma_2)(S_{1R} - S_{2R} - S_{1L} + S_{2L})\end{aligned} \tag{A.4}$$

Therefore $M^2 \geq 0$ for $\gamma_1 = \gamma_2$, i.e. when $B_1 = B_2$.

In general for $B_1 \neq B_2$ there can be tachyons. In particular, consider the case $B_2 = 0$. Eq. (A.2) can be written as

$$\begin{aligned}\alpha' M^2 = \frac{(mR)^2}{\alpha'} + \frac{\alpha'}{(mR)^2}(mn - \gamma_1(S_{1R} + S_{1L}))^2 + 2(N_R + N_L) \\ +2\gamma_1(l_{1L} + l_{1R} + 1) - 2\gamma_1(S_{1R} - S_{1L})\end{aligned} \tag{A.5}$$

From (A.3) one have $|S_{1R,L}| \leq N_{R,L} + 1$. Take, for example, $n = 0$, $m = 1$, $R^2 = \alpha'$, $N_R = N_L = 0$, $l_{1L} = l_{1R} = 0$, $S_{1R} = 1$, $S_{1L} = -1$. We get

$$\alpha' M^2 = 1 - 2\gamma_1 < 0\,, \qquad \text{for}\ \ 1/2 < \gamma_1 < 1\ .$$

Another interesting tachyonic state, appearing at $B_1 \neq B_2$ in a certain range of the parameters, is a state with $N_L = 0$. We take $N_R = nm$, $N_L = 0$, $l_{1L} = l_{1R} = 0$, $S_{1R} = nm + 1$, $S_{1L} = -1$, $S_{2R} = 0$, $S_{2L} = 0$, obtaining

$$\alpha' M^2 = \left(\frac{mR}{\sqrt{\alpha'}} + \frac{\sqrt{\alpha'}n}{R}(1 - \gamma_1)\right)^2 - 2(\gamma_1 - \gamma_2)\ . \tag{A.6}$$

It follows that M^2 can be negative only if

$$(B_1 - B_2) > \frac{mR}{2\alpha'}\ .$$

Setting, for instance, $m = 1$, $\gamma_1 = 1 - 1/n_0$, $B_2 = 0$, $R = \sqrt{\alpha'}$, one finds that M^2 becomes negative for $n_0 > 1 + n + \sqrt{(n+1)^2 + n^2}$.

Note the difference with the true BPS case which occurs for $B_1 = B_2$, where M^2 in (A.6) becomes a perfect square.

B. Rotating folded string

Another class of widely studied quantum string states represents strings which are folded and rotate in the non-compact plane r_1, φ_1. In addition, we will consider the case when it is wrapped in the compact dimension y with winding $m \geq 0$ and zero momentum n. The corresponding ansatz is

$$t = \kappa\tau \ , \quad r_1 = r_1(\sigma) \ , \quad \varphi_1 = \omega\tau \ , \quad y = mR\sigma \, , \tag{B.1}$$

and $r_2 = \varphi_2 = 0$. We will consider this string in the model (3.2). It corresponds to a quantum state with

$$N_R = N_L = N \, , \qquad J_{1R} = J_{1L} = N \, , \qquad J_{2R} = J_{2L} = 0 \, , \tag{B.2}$$

which gives

$$\alpha' M^2 = 4N + \alpha' \left(\frac{mR}{\alpha'} - B_1 J_1 \right)^2 \, , \tag{B.3}$$

where we assume $B_1 > 0$. We will now show that the classical energy of the solution (B.1) reproduces this formula.

The Nambu-Goto Action is $S = \frac{1}{2\pi} \int d\tau d\sigma L$ where the Lagrangian L is

$$L = \frac{1}{\alpha'\Lambda} \sqrt{(\kappa^2\Lambda - r_1^2\omega^2)\big((dr_1/d\sigma)^2\Lambda + m^2R^2\big)} + \frac{B_1 r_1^2}{\alpha'\Lambda}\omega mR \, . \tag{B.4}$$

We can formally consider σ to play the role of "time" and solve for $r_1(\sigma)$ by using the Hamiltonian formalism, where we define a "conjugate momentum" and the "Hamiltonian" by

$$p(\sigma) = \alpha' \frac{\partial L}{\partial(dr_1/d\sigma)} \, , \qquad H(p, r_1) = \frac{1}{\alpha'} p \frac{dr_1}{d\sigma} - L \, . \tag{B.5}$$

We call for short:

$$f \equiv \frac{\kappa^2 + (\kappa^2 B_1^2 - \omega^2) r_1^2}{1 + B_1^2 r_1^2} \, , \qquad g \equiv \frac{m^2R^2}{1 + B_1^2 r_1^2} \, , \qquad A \equiv -\alpha' H(p, r_1) \, . \tag{B.6}$$

The important point is that A is constant. We find

$$\begin{aligned} p^2 &= \frac{(dr_1/d\sigma)^2}{(dr_1/d\sigma)^2 + g} f \, , \\ \sqrt{\frac{fg^2}{(dr_1/d\sigma)^2 + g}} &= \sqrt{(f - p^2)g} = A - \frac{B_1 r_1^2 mR}{1 + B_1^2 r_1^2}\omega \, , \end{aligned} \tag{B.7}$$

giving

$$\left(\frac{dr_1}{d\sigma} \right)^2 = g \frac{fg - \left(A - \frac{B_1 r_1^2 mR}{1 + B_1^2 r_1^2}\omega \right)^2}{\left(A - \frac{B_1 r_1^2 mR}{1 + B_1^2 r_1^2}\omega \right)^2} \, . \tag{B.8}$$

Therefore $fg - (A - \frac{B_1 r_1^2 mR}{1 + B_1^2 r_1^2}\omega)^2 \geq 0$, which is satisfied if

$$0 \leq r_1^2 \leq r_M^2 \, , \qquad r_M^2 \equiv \frac{\kappa^2 m^2 R^2 - A^2}{(mR\omega - AB_1)^2} \, . \tag{B.9}$$

Note that (B.7) holds for arbitrary r_1 and therefore it implies that

$$A \geq 0 \, , \quad A - B_1 r_1^2 (mR\omega - AB_1) \geq 0 \, , \tag{B.10}$$

so that

$$\left|\frac{A}{mR\omega - AB_1} - B_1 r_1^2\right| = \frac{A - B_1 r_1^2(mR\omega - AB_1)}{|mR\omega - AB_1|} \ . \tag{B.11}$$

Some useful formulae, following from rewriting what said above, are:

$$p = \sqrt{f - \frac{1}{g}\left(A - \frac{B_1 r_1^2 mR\omega}{1 + B_1^2 r_1^2}\right)^2} = \frac{|mR\omega - AB_1|}{mR}\sqrt{r_M^2 - r_1^2}\,, \tag{B.12}$$

$$\frac{dr_1}{d\sigma} = \pm\frac{mR\sqrt{r_M^2 - r_1^2}}{|\frac{A}{mR\omega - AB_1} - B_1 r_1^2|} \ . \tag{B.13}$$

The energy of the state is

$$\begin{aligned} E &= \frac{1}{2\pi}\int d\sigma \frac{\partial L}{\partial \kappa} \\ &= \frac{\kappa}{2\pi\alpha'}\int d\sigma \sqrt{\frac{(dr_1/d\sigma)^2 + g}{f}} \\ &= \frac{\kappa}{2\pi\alpha'}\int d\sigma \frac{dr_1}{d\sigma}\frac{1}{p} \\ &= \frac{2n\kappa}{\pi\alpha'}\frac{mR}{|mR\omega - AB_1|}\int_0^{r_M}\frac{dr_1}{\sqrt{r_M^2 - r_1^2}} \\ &= \frac{n\kappa}{\alpha'}\frac{mR}{|mR\omega - AB_1|} \ . \end{aligned} \tag{B.14, B.15}$$

Here we have used (B.13) with the + sign when r grows from 0 to r_M and we have assumed that the string is folded n times; a factor 4 appears because for $n = 1$ any point r_1 of the string is obtained 4 times as σ goes from 0 to 2π.

We have still to require, using (B.13) and (B.11),

$$\begin{aligned} 2\pi = \int_0^{2\pi} d\sigma &= \frac{4n}{|mR\omega - AB_1|}\int_0^{r_M} dr_1 \left|\frac{A - B_1 r_1^2(mR\omega - AB_1)}{mR\sqrt{r_M^2 - r_1^2}}\right| \\ &= \frac{2\pi n}{mR}\left[\frac{A}{|mR\omega - AB_1|} - \epsilon B_1 \frac{r_M^2}{2}\right] , \end{aligned} \tag{B.16}$$

where $\epsilon \equiv \mathrm{sign}(mR\omega - AB_1)$. The formula (B.16) together with (B.9) gives

$$\frac{\kappa^2 m^2 R^2}{(mR\omega - AB_1)^2} = r_M^2 + \frac{A^2}{(mR\omega - AB_1)^2} = r_M^2 + \left(\frac{mR}{n} + \epsilon\frac{B_1 r_M^2}{2}\right)^2 , \tag{B.17}$$

and therefore

$$E = \frac{n}{\alpha'}\sqrt{r_M^2 + \left(\frac{mR}{n} + \epsilon\frac{B_1 r_M^2}{2}\right)^2} \ . \tag{B.18}$$

JHEP08(2007)047

Finally, one can express r_M^2 in terms of the angular momentum J_1:

$$J_1 = \frac{1}{2\pi}\int d\sigma \frac{\partial L}{\partial \omega} = \frac{1}{2\pi\alpha'}\int d\sigma \left(\frac{-\omega\, r_1^2}{1+B_1^2 r_1^2}\sqrt{\frac{(dr_1/d\sigma)^2+g}{f}} + \frac{B_1 m R r_1^2}{1+B_1^2 r_1^2}\right) \tag{B.19}$$
$$= \frac{4n}{2\pi\alpha'|mR\omega - AB_1|}\int_0^{r_M} dr_1\, \frac{-\omega r_1^2 mR + B_1 r_1^2\big(A - B_1 r_1^2 (mR\omega - AB_1)\big)}{(1+B_1^2 r_1^2)\sqrt{r_M^2 - r_1^2}}$$
$$= -\epsilon \frac{n}{2\alpha'} r_M^2\ .$$

Thus

$$\alpha' E^2 = 2n|J_1| + \alpha' \left(\frac{mR}{\alpha'} - B_1 J_1\right)^2 , \tag{B.20}$$

in agreement with (B.3).

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